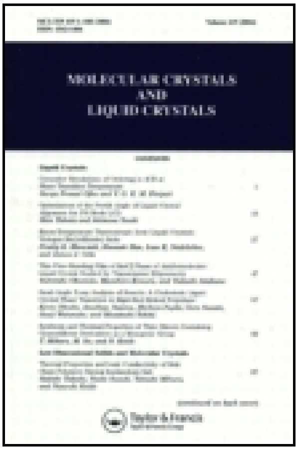

Molecular Crystals and Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl16

Depinning of CHARGE-DENSITY WAVES BY TUNNELING

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Version of record first published: 14 Oct 2011.

To cite this article: John Bardeen (1982): Depinning of CHARGE-DENSITY WAVES BY TUNNELING, Molecular Crystals and Liquid Crystals, 81:1, 1-15

To link to this article: http://dx.doi.org/10.1080/00268948208072546

Mol. Cryst. Liq. Cryst., 1982, Vol. 81, pp. 1–15
0026-8941/82/8101-0001$06.50/0

(Proceedings of the International Conference on Low-Dimensional Conductors, Boulder, Colorado, August 1981)

DEPINNING OF CHARGE-DENSITY WAVES BY TUNNELING

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Received for publication September 2, 1981

A semiconductor model has been used to calculate the current associated with motion of CDW's by Zener tunneling through a small pinning gap. Introduction of a correlation length gives a threshold field for the tunneling current. Application of Tucker's theory gives a scaling relation between the dc and ac conductivities. There is an additional contribution to $\sigma(\omega)$ from oscillations of the pinned wave. Excellent agreement with experiment is obtained for both transitions in $NbSe_3$ and for the commensurate transition in TaS_3. However, the model with Tucker's theory fails to account for effects of combined ac and dc fields. Reasons for the failure and possible ways to develop a satisfactory theory will be discussed.

I. INTRODUCTION

The remarkable field and frequency dependence of the conductivity of $NbSe_3$ below the two Peierls transitions at T_1 = 142 K and T_2 = 59 K was discovered by P. Monceau et al.[1,2] in 1976. The resistivity peaks below these transitions are nearly wiped out at microwave frequencies and at dc fields of a few volts/cm below T_1 and a few tenths volts/cm below T_2. They found that the field dependent conductivity could be expressed approximately in the form

$$\sigma \equiv I/E = \sigma_a + \sigma_b \exp(-E_0/E) , \qquad (1)$$

where σ_a is the low field conductivity and $\sigma_a + \sigma_b$ is approximately that expected if the charge-density waves (CDW's) were not present. X-rays studies[3] showed that the

CDW's are present with undiminished amplitude at fields large enough to wipe out more than half of the resistivity peaks.

The picture is that the CDW's are weakly pinned and can be released to move freely at high fields. When moving, they transport electrons by the Fröhlich mechanism[4] and do not add to the resistivity. We shall show later why the conductivity from unpinned freely moving CDW's is approximately that expected in the absence of the CDW's.

The electric field dependence of Eq. (1) is that expected from a tunneling mechanism. It was first suggested that the CDW's become depinned by Zener tunneling of electrons across a small semiconducting gap. However it was found that the gap required is much less than kT, so this mechanism for depinning did not appear to be plausible. To try to rescue the Zener tunneling theory, I suggested[5] in 1979 that the tunneling applies only to motion of the sea of electrons condensed in the CDW and not to individual electrons. The energy involved is not that of individual electrons, but that associated with the coherent motion of the CDW; including the kinetic energy of the ions. I gave arguments to suggest that the energy gap, ε_g, should be equal to $\hbar\omega_p$, where ω_p is the frequency of oscillation of the CDW about the pinning positions. A uniform gap was assumed such as might arise from commensurability pinning. The theory also should apply to pinning by collective action of large numbers of weak impurities (e.g. Ta) rather than a few strong ones (e.g. Ti).

An alternative theory for depinning based on a classical model of sliding friction was given by Lee and Rice. They predicted that E_o should vary as the square of the impurity concentration, c_i, a prediction that was later verified by experiment.[7] The tunneling model also gives this result.

Another approach, based on an overdamped oscillator model that also relates depinning in an electric field with pinning frequencies, is being discussed by Portis and by Grüner at this meeting. This model has been worked out in some detail by Grüner, Zawadowski and Chaikin.[8]

It was shown by Fleming and Grimes[9] that (1) does not apply for small values of E and that there is a threshold field, E_T, that must be exceeded before there is a conductivity change. Fleming[10] showed that the data for $E>E_T$ could be fitted by the empirical expression:

$$\sigma = \sigma_a + \sigma_b(1-(E/E_T))\exp[E_o/(E-E_T)], \quad E > E_T . \tag{2}$$

In the summer of 1980 there appeared excellent quantitative data on the field dependence of conductivity of $NbSe_3$ below T_2 by J. Richard and P. Monceau[11] and by J. W. Brill et al.,[7] confirming the existence of a threshold field. The data seemed too consistent to be accounted for by a sliding friction model. In reviewing the tunneling theory, I found that a threshold field would be introduced if there is a finite correlation length, L, for the CDW such that the field E must be applied within L to be effective in accelerating the wave. With this revision,[12] for $E > E_T$,

$$\sigma = \sigma_a + \sigma_b(1-(E/E_T))\exp(-E_o/E_T) \quad , \tag{3}$$

and when L is small compared with $4\hbar v_F/\pi\varepsilon_g$ (twice the Pippard coherence distance), $E_o = E_T$, so there is only one parameter rather than two. This revised theory gave an excellent fit to the experimental data on $NbSe_3$ below T_2.

A theory developed by J. Tucker[13] for photon-assisted tunneling across superconductor-insulator-superconductor (SIS) tunnel junctions was used to account for the frequency dependence of the conductivity. This theory predicts a scaling between frequency and field dependence of the tunneling contribution

$$\sigma_t(\omega/\omega_T) = \sigma_{dc}(E/E_T) \quad . \tag{4}$$

Except for a small excess near threshold, Eq. (4) gave a good quantitative fit to the only data available at the time, that of Grüner et al. on $NbSe_3$ taken at one temperature below T_2.

During the past few months Grüner, Clark and associates[15] at UCLA have made extensive measurements of the field and frequency dependence of the conductivity of both CDW phases of $NbSe_3$ and of TaS_3. They find excellent quantitative agreement with the scaling relation at high fields and frequencies ($E>2E_T$), but observe an excess contribution to $\sigma(\omega)$ at low frequencies ($\omega \sim \omega_T$ and below) that increases with decreasing temperature. There is no threshold for frequency dependence as there is for field dependence. This additional contribution can be attributed to absorption by oscillations of the pinned CDW about the pinning frequency, ω_p.

They have also made numerous experiments with combined ac and dc voltages applied to try to check other predictions of Tucker's theory; in particular to find evidence for photon-assisted tunneling. They find that the interaction between ac and dc fields is much less than predicted and no evidence for photon-assisted tunneling. In the following we shall discuss reasons for this failure of the theory when ac and dc fields are combined even though it fits the data for pure ac or pure dc.

In Sec. II I discuss Fröhlich conduction by freely moving (unpinned) CDW's and show that the conductivity is about that expected in the absence of the CDW's. Section III gives a brief review of the semiconductor model for depinning for dc applied voltages and Sec. IV the application of Tucker's theory to account for the ac conductivity. The reasons why Tucker's theory does not apply to combined ac and dc voltages is discussed in Sec. V and Sec. VI gives concluding remarks and discusses areas for further theoretical development.

II. FRÖHLICH CONDUCTION BY FREELY MOVING CDW'S

In this section, I shall try to make plausible why the limiting conductivity for high fields or high frequencies is about that expected in the absence of CDW's. According to Fröhlich's picture, when a CDW is moving with drift velocity v_d, the Peierls gaps appear at the moving 1D Fermi surface, at k_F+q and $-k_F+q$, where $\hbar q = mv_d$, and m is the band mass.

In a simple model, the charge density associated with the moving CDW is

$$\rho(x) = \rho(0)\cos 2k_F(x-v_d t) \tag{5}$$

so that the fundamental frequency, presumably that observed in narrow-band noise, is $\omega_d = 2k_F v_d$. This may also be regarded as the frequency of the macroscopically occupied phonon state of wave vector $2k_F$.

Possible transitions within the CDW occur by scattering of electrons in the state k_F+q at one boundary of the Fermi sea to $-k_F+q$ at the opposite boundary with emission of a $2k_F$ phonon or the reverse:

$$k_F+q \leftrightarrow -k_F+q + 2k_F \text{ (phonon)}. \tag{6}$$

The energy is balanced, since

$$\varepsilon_{k_F+q} = \varepsilon_{-k_F+q} + \hbar\omega_{ph} \tag{7}$$

with

$$\hbar\omega_{ph} = 2\hbar k_F v_d = 2\hbar v_F q \ . \tag{8}$$

One may regard the CDW as resulting from a rapid momentum exchange between electrons and lattice. If $P_e = n\hbar q$ is the net crystal momentum of the electrons and $P_L \stackrel{e}{=} (N_{2k_F} - N_{-2k_F})2\hbar k_F$ that of the lattice, in steady state (and the absence of other relaxation processes),

$$P_e/\tau_{eL} = P_L/\tau_{Le} \tag{9}$$

where τ_{eL} is the relaxation time for electrons to create $2k_F$ phonons and τ_{Le} that for $2k_F$ phonons to scatter electrons from $-k_F+q$ to k_F+q. In a simple model, the ratio

$$P_L/P_e = \tau_{Le}/\tau_{eL} \equiv M_F/m \equiv \alpha \tag{10}$$

where M_F is the Fröhlich mass associated with ion motion and m is the band mass. I define α to be the ratio $M_F/m \sim 10^3$ so that $P_L = \alpha P_e$.

If there were no other scattering processes, the motion of the CDW would persist in time as a supercurrent. The system would then be a 1D model of a superconductor, as suggested by Fröhlich. Actually, in $NbSe_3$, electrons may be scattered to other parts of the Fermi surface not involved in the CDW and $2k_F$ phonons may lose their momentum to electrons or to other phonon modes. In TaS_3, the entire Fermi surface is involved, so the only possibility is decay of $2k_F$ phonons to other modes. We define τ_e to be the relaxation for electrons other than $2k_F$ phonons and τ_L that for $2k_F$ phonons other modes. Presumably $\tau_e >> \tau_{eL}$, $\tau_L >> \tau_{Le}$, since the most rapid process is exchange of momentum within the CDW.

In an electric field E, the equations of motion for the electrons and phonons are:

$$\frac{dP_e}{dt} + \frac{P_e}{\tau_e} + \frac{P_e}{\tau_{eL}} = neE + \frac{P_L}{\tau_{Le}} \tag{11}$$

$$\frac{dP_L}{dt} + \frac{P_L}{\tau_L} + \frac{P_L}{\tau_{Le}} = \frac{P_e}{\tau_{eL}} \ . \tag{12}$$

From these equations

$$\frac{d(P_e+P_L)}{dt} + \frac{P_L}{\tau_L} + \frac{P_e}{\tau_e} = neE \ . \tag{13}$$

With $P_L = P_e$ and $P_e = nhq$, an equation of motion for q may be derived:

$$hdq/dt + q/\tau^* = e^*E \tag{14}$$

where $e^* = e/(1+\alpha) \sim 10^{-3}e$ and

$$\frac{1}{\tau^*} = \frac{1}{1+\alpha}\left\{\frac{\alpha}{\tau_L} + \frac{1}{\tau_e}\right\} \ . \tag{15}$$

The dc conductivity of the unpinned CDW is then

$$\sigma_{dc} = nee^*\tau^*/m = ne^2\tau_{eff}/m \ , \tag{16}$$

where $\tau_{eff} = [(\alpha/\tau_L) + (1/\tau_e)]^{-1}$ is about that expected in the absence of CDW's. In the case of TaS_3, there are CDW fluctuations above T_p, so the relaxation process may occur via $2k_F$ phonons as it is below T_p. The high frequency conductivity will be resistive and equal to σ_{dc} as long as $\omega\tau_{eff} << 1$.

III. SEMICONDUCTOR MODEL FOR DEPINNING

Even though there may be a Peierls gap and thus no quasi-particle excitations at low temperatures, in the absence of pinning there is no gap for changing the displacement, q, the Fermi sea and thus the velocity, v_d, of the CDW. The displacement q changes in an electric field as would that of a Fermi sea of particles of effective charges e* and relaxation time τ*.

Pinning can be represented by a small pinning gap at the Fermi surface, as illustrated in Fig. 1. The Fermi sea is now represented by filled states below the gap. In an electric field, the particles would drift according to $\hbar dk/dt = e^*E$, but there would be no current since the lower band remains filled. This corresponds to total reflection when $k \rightarrow k_F$.

Current can flow if in an electric field particles can go from the lower band to the upper band, as indicated in the dotted line. In the tunneling model, they do this by Zener tunneling. The probability that particles reaching k_F tunnel to the upper band is $P = \exp(-E_0/E)$, where E_0 is

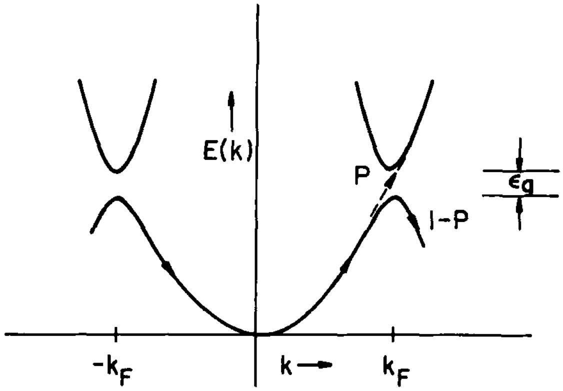

FIGURE 1 Semiconductor model for Fermi sea of electrons condensed in a pinned CDW. The lower band is occupied and the upper unoccupied. In an electric field current flows and the CDW becomes depinned by Zener tunneling through the small pinning gap. Since the Peierls gaps move with the Fermi sea they do not need to be considered in the depinning process.

given by the Zener expression $\varepsilon_g/(2\xi_o e^*)$ and $\xi_o = 2\hbar v_F/\pi\varepsilon_g$ is the Pippard coherence distance, a measure of the distance over which the field is effective in accelerating the Fermi sea. Thus $2\xi_o e^*E_o = \varepsilon_g$.

Thus far the model leads to Eq. (1) with an expression for E_o in terms of the gap. Arguments were given to show that $\varepsilon_g = \hbar\omega_p$, and thus to relate E_o with the pinning frequency.

A threshold field was introduced through a finite correlation length, L, across which the field must be applied to be effective in accelerating the Fermi sea. It is presumed that L represents the distance over which the phase of the CDW is correlated. Illustrated in Fig. 2 is a Zener diagram showing the energy gap tilted in space by the electric field with slope e*E. Tunneling takes place over horizontal lines of constant energy. For tunneling to be effective over a length L, one must go from a filled state below the gap to an empty state above. No tunneling is possible unless $e^*EL > \varepsilon_g$, which gives a threshold field $E_T = \varepsilon_g/e^*L$. Further the distance over which tunneling can occur is reduced by a factor $(1-(E/E_T))$ for $E>E_T$.

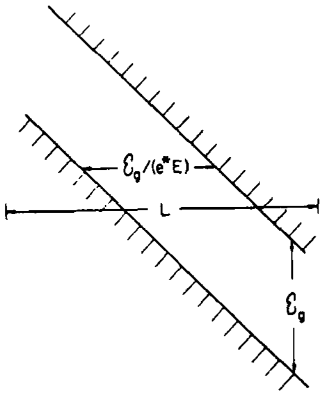

FIGURE 2 Zener diagram showing energy bands near the energy gap tilted in space by an effective potential energy e*Ex. If there is a correlation length L for the CDW such that an electric field must act within L to be effective in accelerating the wave; no tunneling can occur unless $e^*EL > \varepsilon_g$ and the distance over which tunneling occurs is reduced to $L-(\varepsilon_g/e^*E)$.

The Zener expression assumes that tunneling occur over a distance $2\xi_o$. If $L \ll 2\xi_o$, one must replace $2\xi_o$ by L, which makes $E_o = E_T$. In this limit there is only one parameter in the theory instead of two. The conductivity is given by $\sigma = \sigma_a + \sigma_b P(x)$, where $x = E/E_T$ and

$$P(x) = (1-x^{-1})\exp(-x^{-1}) \quad . \qquad (17)$$

As seen in Fig. 3, reproduced from ref. 5, the data of Richard and Monceau for the CDW below T_2 in $NbSe_3$ are in excellent agreement with Eq. (17). More generally, $E_o = E_T(1+(L/2\xi_o))$. For temperatures just below T_P, E_o is further enhanced by a factor n/n_c, where n_c is the density of electrons condensed in the CDW and which varies as $(1-(T/T_P))^{1/2}$ as $T \to T_P$.

For weak impurities, the pinning energy and thus the pinning gap is proportional to the impurity concentration, c_i, and L and ξ_o are inversely proportional to c_i. Thus E_T and E_o are both proportional to c_i^2, in agreement with experiments of Brill et al.

Subsequent experiments, particularly by Grüner[15] et al. at UCLA, have shown that (3) is in excellent quantitative

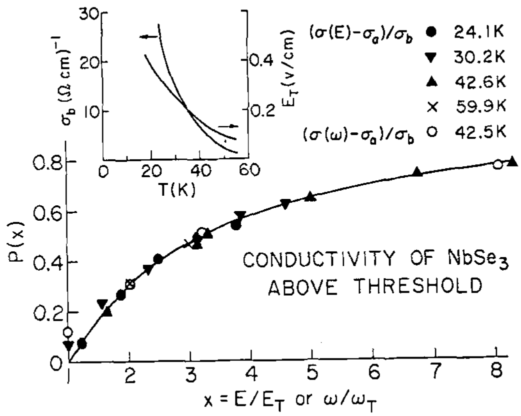

FIGURE 3 Conductivity of $NbSe_3$ plotted in reduced units compared with P(x) from Eq. (17). The dc data are from Richard and Monceau[11] and the ac from Grüner et al.[14]

agreement for the dc nonlinear conductivity of both phases of $NbSe_3$ and also for depinning of the commensurate CDW in the TaS_3. With appropriate choice of parameters, Fleming's empirical expression, Eq. (2), also gives a reasonable fit, since for $E-E_T \ll E_T$, the exponential is very small. Values of L are typically of the order of 10μm for "pure" specimens and ε_g is of the order 10^{-19} ergs in $NbSe_3$ and somewhat larger in TaS_3.

IV. FREQUENCY DEPENDENCE OF CONDUCTIVITY - SCALING

The success of the semiconductor model of tunneling for dc conduction suggested applying Tucker's theory of photon-assisted tunneling to acount for the frequency dependence of the ac conductivity. Tucker derived equations for the current from an applied voltage $V_0 + V_1\cos\omega t$ across a tunnel junction. For small signal ac, $eV_1 < \hbar\omega$, Tucker's expressions[13] for the ac current, $I_1(\omega)\cos\omega t$, and the change in dc current from the ac voltage are:

$$I_1(\omega) = (eV_1/2\hbar\omega)[I_o(V_0+(\hbar\omega/e)) - I_o(V_0-(\hbar\omega/e))] \quad (18)$$

$$\Delta I_{dc} = (eV_1/2\hbar\omega)^2[I_o(V_0+(\hbar\omega/e) - 2I_o(V_0) + I_o(V_0-(\hbar\omega/e))] \ . \quad (19)$$

Here $I_o(V)$ is the dc current for an applied voltage V. To apply to the semiconductor model, the voltage V is taken to be that across a correlation length L and the charge e is replaced by e*. In the limit $V_0 = 0$, Eq. (18) gives

$$I_1(\omega) = (e^* V_1/\hbar\omega) I_o(\hbar\omega/e^*) \ . \tag{20}$$

The ac conductivity $\sigma(\omega)$ is equal to the dc conductivity (defined as I/E) for a field $E = \hbar\omega/e^*L$. This gives the scaling relation, Eq. (4), for the tunneling contribution to $\sigma(\omega)$. The energy gap, $\varepsilon_g = e^*E_T L = \hbar\omega_T$.

At the time the paper[12] was written (1980), the only data on $\sigma(\omega)$ were a few points[14] taken at one temperature, 42 K, on $NbSe_3$. These are plotted in reduced units in Fig. 3 and give an excellent fit to the function P(x), where $x = \omega/\omega_T$, except for the point at threshold, $\omega = \omega_T$. Later detailed measurements of both ac and dc conductivity on the same specimens by Grüner, Zettl and Clark[15] showed that scaling holds very well for $\omega > \omega_T$ for both the upper and lower transition in $NbSe_3$ and for $\omega > \omega_T$ for TaS_3. Further, it was found that there is an additional contribution to $\sigma(\omega)$ from absorption by the pinned CDW and that this additional contribution extends down to $\omega \to 0$ so that there is no threshold frequency. The contribution is small near T_P but increases with decreasing temperature.

The additional contribution, $\sigma^P(\omega)$, derived from the phenomenological equation of motion of the pinned wave,

$$d^2x/dt^2 + \Gamma(dx/dt) + \omega_p^2 x = (e^*/m)E(t) \ , \tag{21}$$

is

$$\sigma^P(\omega) = \sigma^P(\omega_p) \left\{ \frac{\Gamma^2\omega^2}{(\omega_p^2-\omega^2)^2 + \Gamma^2\omega^2} \right\} \ . \tag{22}$$

For $NbSe_3$, a good fit is obtained by taking $\omega_p = \omega_T$, in agreement with taking an energy gap equal to $\hbar\omega_p$. However it is found that for TaS_3, ω_p is larger than ω_T, with ω_p increasing with decreasing temperature. The difference between $NbSe_3$ and TaS_3 may arise from the difference between incommensurate and commensurate pinning.

Last fall when the theory was being developed, it was hoped that one could use photon-assisted tunneling as described by Eq. (19) in linear-chain compounds to detect an ac signal as is done very successfully with SIS tunnel junctions. In the latter, it is possible to take the dc bias V_0 below threshold and use the quantum energy, $\hbar\omega/e$, to bring $I_0(V_0 + (\hbar\omega/e))$ above threshold. The tunnel junction then acts as a photon detector. Experiments of Grüner et al.[15] soon showed that this failed in $NbSe_3$. When biased such that $V_0 + V_1$ is below threshold, no detected dc current was observed even when ω was well above threshold. A change in dc current was observed only at low frequencies, $\omega << \omega_T$, and only when $V_0 + V_1$ was above threshold. In this case the combined voltage, $V_0 + V_1 \cos\omega t$, can be considered to be a slowly varying dc and the response is that to be expected classically. As ω is increased for constant V_1, ΔI_{dc} drops rapidly and is very small, less than a few percent of that expected classically, when $\omega \sim \omega_T$. Other experiments, for example on the effect of a dc bias on $\sigma(\omega)$, also gave negative results.

Thus a theory that is in apparent agreement with experiment for pure dc or pure ac fails when ac and dc are combined. The probable reasons for this failure are discussed in the next section.

V. FAILURE OF THE SEMICONDUCTOR MODEL FOR COMBINED DC AND AC VOLTAGES

The failures of the semiconductor tunneling model with Tucker's theory to account for observed effects of the interaction between dc and ac applied voltages may be summarized as follows:

1. There is no indication of a change in dc current, ΔI_{dc}, when $V_0 + V_1 < V_T$ and $V_0 + (\hbar\omega/e^*) > V_T$ as expected from photon-assisted tunneling.
2. The ac conductivity in the presence of a dc field, E, $\sigma(\omega,E)$, does not increase as expected with increasing field when $EL + \hbar\omega/e^*$ becomes greater than V_T.
3. Even when $V_0 > V_T$, ΔI_{dc} is much smaller than expected except for very low frequencies, $\omega << \omega_T$.
4. The theory does not account for effects of large amplitude applied ac voltages, including resistivity peaks observed when the applied frequencies is close to the fundamental phonon frequency, $\omega \simeq \omega_{ph} = 2k_Fv_d$, or to harmonics or subharmonics of ω_{ph}. These frequencies are observed to be proportional to the current density, $n_c ev_d$, associated with CDW motion.

It is believed that the difficulties are due to the semiconductor model that does not contain enough of the physics of charge-density waves and was designed originally to account for depinning in an electric field. The only way CDW's enter the model is through the replacement of e by e*. This accounts for the rapid sharing of electron momentum with that of ion motion; when an electron in the CDW condensate is accelerated it almost immediately gives up most of the momentum gain to the lattice. However, the model does not include other aspects of CDW's, such as the frequency of the macroscopically-occupied phonon state, ω_{ph}. It is believed that Tucker's theory should apply to the model and that the difficulty is with the model.

In Tucker's theory, a quantum frequency from the dc voltage across the correlation L, $\omega_o = e^*V_0/\hbar$ is combined with the applied ac frequency, ω. This represents the frequency associated with the energy difference between electrons tunneling from one side of L and holes from the opposite. In the semiconductor model, the energy gained by an electron from the voltage V_0 across L is e^*V_0, and this frequency is present in the wave function for the amplitude of the tunneling electrons. However, when CDW's are present, the fundamental frequency is $\omega_{ph} = \omega_o P(E)$, not ω_o. This is the frequency that is proportional to the actual dc current flowing.

Schematically, the wave function may be written $\Psi = (1-P)^{1/2}\Psi_0 + P^{1/2}\Psi_1$, where Ψ_1 is the part associated with tunneling across the gap and corresponds to the wave function of an unpinned wave. The frequency ω_o appears in Ψ_1 in the semiconductor model and is the phonon frequency for an unpinned CDW. One cannot consider a CDW as having a probability (1-P) of no flow ($\omega_{ph} = 0$) and of P of moving freely ($\omega_{ph} = \omega_o$). The actual current flow and frequency are reduced by the probability P, to give the phonon frequency $\omega_{ph} = \omega_o P$. Thus when CDW's are present, one is not allowed to combine probability amplitudes, only probabilities.

To a first approximation then, one may determine the effect of an ac voltage superimposed on a dc by first finding the motion of the CDW from the applied dc voltage and then determine the response of the moving CDW to the superimposed ac voltage. The latter is essentially a classical calculation. At very low frequencies the ac may be regarded as a slowly varying dc, so that the voltages may be added to determine the response. This is in accord with experiments with large amplitude ac applied voltages combined with dc. Low frequency implies $\omega < \omega_{ph}$ and in practice this implies $\omega << \omega_T$. For values of ω is of the order of ω_T and higher, there is little interaction between ac and dc applied voltages.

True nonlinear effects associated with CDW motion do give an interaction between ac and dc for large amplitude ac voltages, with the amplitude required for a given response increasing with increasing frequency. We still do not have a satisfactory quantitative theory to calculate the nonlinear response, but classical models such as those of Grüner et al. and of Portis should give the correct qualitative behavior.

VI. CONCLUSIONS

The tunneling theory gives excellent quantitative agreements with experiment for both field and frequency dependence of the conductivity of both CDW phases of $NbSe_3$ and of TaS_3. There is strong evidence for tunneling across a correlation length of the order of 10 microns across an energy gap for electrons of the order of 10^{-19} ergs. The value of L is consistent with electron microscopy measurements of CDW motion in $NbSe_3$ by Fung and Steeds.[16] They find transverse coherence distances of the order of 2-300 Å. The total volume occupied by the cohrent motion is of the order of 10^{-14} cm^3 and contains about 10^7 electrons. There is an additional contribution to the ac conductivity from absorption by oscillations of the pinned wave.

There is no photon-assisted tunneling from combined dc and ac voltages. Except at frequencies so low that the ac can be considered to be a slowing varying dc, the interaction between dc and ac voltages is very weak. No satisfactory theory exists for nonlinear effects observed for large amplitude ac voltages combined with dc.

It is suggested that experiments with pulsed dc voltages of varying duration and amplitude, such as those of Ong and Tessema,[17] would be helpful in understanding the nonlinear effects associated with large amplitude ac voltages. It may be expected that in the immediate future theoretical explanations will involve a combination of quantum and classical concepts such as those us to describe the frequency dependence of the conductivity with quantum tunneling and a classical treatment of absorption by the pinned wave. Hopefully it will eventually be possible to develop a microscopic theory in which the quantum aspects are treated by a more realistic model. Any such theory would have to include the effects of damping on the tunneling probability.

Experiments on steps corresponding to resistivity peaks in the V(I) characteristics of $NbSe_3$ with a superimposed ac current as observed by Monceau, Richard and Renard[18] provide valuable information on the interaction between dc and ac currents. The peaks are observed when the applied frequency, ω , resonates with the fundamental frequency, ω_{ph}, or one of its harmonics or subharmonics. The locking in of ω_{ph} with ω tends to keep the current from changing with the dc voltage. The resonant resistivity peaks are very narrow, suggesting that the various CDW domains tend to lock in the same frequency and current density. This could be induced by exchange of $2k_F$ phonons between domains as well as by the electromagnetic field.

ACKNOWLEDGMENTS

The author is indebted to G. Grüner, J. R. Tucker and P. D. Coleman for many stimulating discussions and correspondence regarding CDW phenomena. He is most grateful for the opportunity to spend several weeks earlier this summer at the Centre de Recherches sur les Très Basses Température; C.N.R.S., Grenoble, where some of the ideas expressed in this talk were formulated with frequent discussions with P. Monceau, J. Richard, M. Papoular and M. Renard.

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